Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

AS
MATHEMATICS

Unit Further Pure 1

Wednesday 14 June 2017 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 A curve passes through the point $(4, 8)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2x + \sqrt{x}}$$

Use a step-by-step method with a step length of 0.3 to estimate the value of y at $x = 4.6$. Give your answer to four decimal places.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

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QUESTION PART REFERENCE	Answer space for question 1

Turn over ►

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2 The equation $5x^2 + px + q = 0$, where p and q are constants, has roots α and $\alpha + 4$.

(a) Show that $p^2 = 20q + 400$.

[4 marks]

(b) A quadratic equation has roots α^2 and $(\alpha + 4)^2$.

(i) Find this quadratic equation, giving your answer in terms of q.

[3 marks]

(ii) Hence, or otherwise, given that the roots of this quadratic equation are equal, find the value of q.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

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QUESTION PART REFERENCE	Answer space for question 2

3 It is given that $z = \mathrm{i}(1-\mathrm{i})(2+\mathrm{i})$.

(a) Show that z can be expressed in the form $k+3\mathrm{i}$, where k is an integer.

[3 marks]

(b) Hence find the values of the integers m and n such that

$$(z-\mathrm{i})^{*} - mz = n(1+4\mathrm{i})$$

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 3**

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QUESTION PART REFERENCE	Answer space for question 3

4 (a) Find, in terms of c and d, the value of $\int_c^d \frac{1}{2x\sqrt{x}}\,\mathrm{d}x$, where $0 < c < d$.

[3 marks]

(b) Hence show that only one of the following improper integrals has a finite value, and find that value:

(i) $\int_0^9 \frac{1}{2x\sqrt{x}}\,\mathrm{d}x$;

(ii) $\int_9^\infty \frac{1}{2x\sqrt{x}}\,\mathrm{d}x$.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

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QUESTION PART REFERENCE	Answer space for question 4

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5 (a) Find the general solution of the equation

$$\tan\left(2x + \frac{\pi}{2}\right) = \sqrt{3}$$

giving your answer for x in terms of π in a simplified form.

[4 marks]

(b) Use your general solution to find the possible exact values of $\sin 3x - \sin 4x$ given that $\tan\left(2x + \frac{\pi}{2}\right) = \sqrt{3}$.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

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6 An ellipse E_1 has equation $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

(a) Find the area of the rectangle whose vertices are the points of intersection of the horizontal and vertical tangents to the ellipse E_1 .

[2 marks]

(b) The ellipse E_1 can be mapped onto a circle of radius 4 by means of a one-way stretch. Write down the matrix which represents this stretch.

[2 marks]

(c) The ellipse E_1 is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$ to give the ellipse E_2 .

The vertical tangents to E_2 have equations $x = 7$ and $x = -1$.

The equation of E_2 is $x^2 + 4y^2 + px + qy = 3$, where p and q are integers.

(i) Find the value of a .

[2 marks]

(ii) Find the value of p and the possible values of q .

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 6**

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QUESTION PART REFERENCE	Answer space for question 6

7 Use the relevant formulae for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that:

(a) $\sum_{r=1}^{n} \left(r^3 - 3r\right) = \frac{n}{4}(n+a)(n+b)(n+c)$, where a, b and c are integers;

[4 marks]

(b) the sum of the series

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots - (2n)^2 = -n(pn+q)$$

where p and q are integers.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 7**

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QUESTION PART REFERENCE	Answer space for question 7

8 The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

(a) Given that $\mathbf{C} = \begin{bmatrix} 2 & 4 \\ 6 & -2 \end{bmatrix}$ and $\mathbf{C} - 2\mathbf{D} = \mathbf{A}$, find the matrix $\mathbf{D}$.

[2 marks]

(b) Describe fully the single geometrical transformation represented by the matrix $\mathbf{A}$.

[1 mark]

(c) (i) The matrix $\mathbf{B}$ represents an anticlockwise rotation through an **obtuse** angle θ about the origin, where $\sin\theta = \frac{3}{5}$. Find the matrix $\mathbf{B}$.

[2 marks]

(ii) The point $(10,\ 15)$ is mapped onto point P under the transformation represented by $\mathbf{A}$ followed by the transformation represented by $\mathbf{B}$. Find the coordinates of P.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 8**

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9 A curve C has equation

$$y=\frac{2x^2+2x+1}{(x+1)(x-3)}$$

The curve has two stationary points P and Q.

(a) Write down the equations of all the asymptotes of C.

[2 marks]

(b) The line $y=k$ intersects the curve C. Show that $4k^2-3k-1\geqslant 0$.

[5 marks]

(c) **Hence** find the length of the line segment PQ.

(No credit will be given for solutions based on differentiation.)

[7 marks]

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