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# AS **MATHEMATICS**

Unit Further Pure 1

Wednesday 14 June 2017 Morning Time allowed: 1 hour 30 minutes

### **Materials**

For this paper you must have:

the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Instructions

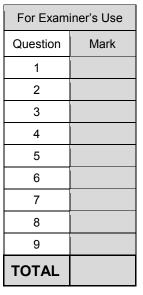
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do not use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information

- The marks for guestions are shown in brackets.
- The maximum mark for this paper is 75.

## Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.





# Answer all questions.

Answer each question in the space provided for that question.

**1** A curve passes through the point (4,8) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2x + \sqrt{x}}$$

Use a step-by-step method with a step length of 0.3 to estimate the value of y at x=4.6. Give your answer to four decimal places.

[5 marks]

QUESTION PART REFERENCE	Answer space for question 1



QUESTION PART REFERENCE	Answer space for question 1
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2		The equation $5x^2 + px + q = 0$ , where $p$ and $q$ are constants, has roots $\alpha$ and $\alpha$	+ 4.
(a)	)	Show that $p^2 = 20q + 400$ .	[4 marks]
(b)	)	A quadratic equation has roots $\alpha^2$ and $(\alpha + 4)^2$ .	
	(i)	Find this quadratic equation, giving your answer in terms of $q$ .	[3 marks]
	(ii)	Hence, or otherwise, given that the roots of this quadratic equation are equal, fi	nd the
		value of $q$ .	[2 marks]
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QUESTION PART REFERENCE	Answer space for question 2
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PART REFERENCE	Answer space for question 2



QUESTION PART REFERENCE	Answer space for question 2
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- 3 It is given that z = i(1-i)(2+i).
  - (a) Show that z can be expressed in the form k+3i, where k is an integer.

[3 marks]

**(b)** Hence find the values of the integers m and n such that

$$(z-i)^* - mz = n(1+4i)$$

[5 marks]

QUESTION PART REFERENCE	Answer space for question 3



PART REFERENCE	Answer space for question 3
REFERENCE	



**4 (a)** Find, in terms of c and d, the value of  $\int_{c}^{d} \frac{1}{2x \sqrt{x}} dx$ , where 0 < c < d.

[3 marks]

**(b)** Hence show that only one of the following improper integrals has a finite value, and find that value:

(i) 
$$\int_0^9 \frac{1}{2x \sqrt{x}} dx$$
;

(ii) 
$$\int_9^\infty \frac{1}{2x \sqrt{x}} \, \mathrm{d}x \, .$$

[3 marks]

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QUESTION PART REFERENCE	Answer space for question 4



**5** (a) Find the general solution of the equation

$$\tan\left(2x + \frac{\pi}{2}\right) = \sqrt{3}$$

giving your answer for x in terms of  $\pi$  in a simplified form.

[4 marks]

(b) Use your general solution to find the possible exact values of  $\sin 3x - \sin 4x$  given that  $\tan \left(2x + \frac{\pi}{2}\right) = \sqrt{3}$ .

[3 marks]

QUESTION PART REFERENCE	Answer space for question 5



QUESTION PART REFERENCE	Answer space for question 5	



- An ellipse  $E_1$  has equation  $\frac{x^2}{16} + \frac{y^2}{4} = 1$ .
  - (a) Find the area of the rectangle whose vertices are the points of intersection of the horizontal and vertical tangents to the ellipse  $E_1$ .

[2 marks]

(b) The ellipse  $E_1$  can be mapped onto a circle of radius 4 by means of a one-way stretch. Write down the matrix which represents this stretch.

[2 marks]

(c) The ellipse  $E_1$  is translated by the vector  $\begin{bmatrix} a \\ b \end{bmatrix}$  to give the ellipse  $E_2$ .

The vertical tangents to  $E_2$  have equations x = 7 and x = -1.

The equation of  $E_2$  is  $x^2 + 4y^2 + px + qy = 3$ , where p and q are integers.

(i) Find the value of a.

[2 marks]

(ii) Find the value of p and the possible values of q .

[4 marks]

QUESTION PART	Answer space for question 6
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- 7 Use the relevant formulae for  $\sum_{r=1}^{n} r^3$ ,  $\sum_{r=1}^{n} r^2$  and  $\sum_{r=1}^{n} r$  to show that:
  - (a)  $\sum_{r=1}^{n} (r^3 3r) = \frac{n}{4} (n+a)(n+b)(n+c), \text{ where } a, b \text{ and } c \text{ are integers};$

[4 marks]

(b) the sum of the series

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \dots - (2n)^2 = -n(pn+q)$$

where p and q are integers.

[4 marks]

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8 The matrix **A** is defined by  $\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ .

(a) Given that 
$$C = \begin{bmatrix} 2 & 4 \\ 6 & -2 \end{bmatrix}$$
 and  $C - 2D = A$ , find the matrix  $D$ .

[2 marks]

(b) Describe fully the single geometrical transformation represented by the matrix **A**. **[1 mark]** 

(c) (i) The matrix **B** represents an anticlockwise rotation through an **obtuse** angle  $\theta$  about the origin, where  $\sin \theta = \frac{3}{5}$ . Find the matrix **B**.

[2 marks]

(ii) The point (10, 15) is mapped onto point P under the transformation represented by  $\mathbf{A}$  followed by the transformation represented by  $\mathbf{B}$ . Find the coordinates of P.

[3 marks]

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9	A curve C ha	is equation

$$y = \frac{2x^2 + 2x + 1}{(x+1)(x-3)}$$

The curve has two stationary points P and Q.

(a) Write down the equations of all the asymptotes of C.

[2 marks]

**(b)** The line y = k intersects the curve C. Show that  $4k^2 - 3k - 1 \ge 0$ .

[5 marks]

(c) Hence find the length of the line segment PQ.

(No credit will be given for solutions based on differentiation.)

[7 marks]

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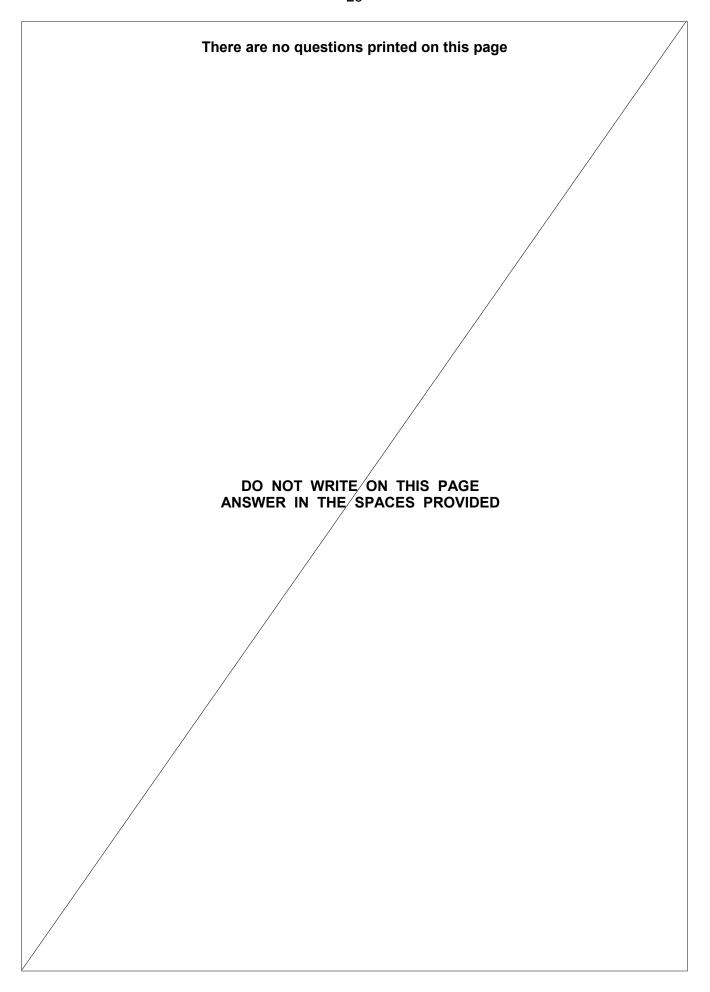


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